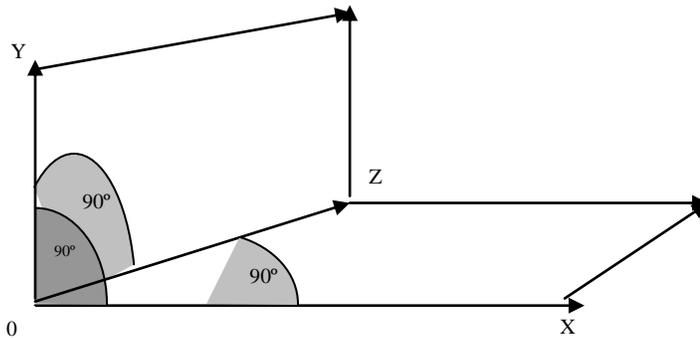


THE FOURTH SPATIAL DIMENSION

Have you seen a film, where the actor is sitting on a couch, the camera is behind him, and he puts a cigarette to his lips, puffs, expels the smoke, then the camera focuses on his hand and blurs the background scene? It presents the world to you as it is from the actor's perspective. You are seeing the world from the visual perspective of the actor. This technique in filmmaking was taken to an extreme in some arty movies in the 30's, where you never see the actor to whom the action is happening to at all. In these films, you hear and see only what the actor hears and sees. In effect, you become the actor/actress. These films never caught on and were abandoned by the early 40's. We, the viewers, want to know what is happening to the actors of a film, not *be* the person experiencing a scene in a film. These cinematic experiments failed, because we watch films to see into the world of others as if they were in a separate dimension we're observing but not part of directly. We are not usually conscious of this underlying mental perspective. It is called *suspension of disbelief* in filmmaking. This aversion to watching a film in first person perspective may change if future developments (like the *Wii* device in gaming) will let us actually interact with a movie.

Everyday life is a visual perspective movie to me. I see everything, minus the milliseconds of my blinking eyes, from my head. It feels to me like I see all things in my visual perspective, though I never see the back of head, nor do I have an idea of my posterior, or back (unless I look at these body parts in a mirror). I am aware of my own body, by a sense of feeling called *touch*. I can feel my chin, as I am now doing. I feel as if I am looking at an external world through my eyes in a persistent sensation. That is to say, my looking out at the world through my eyes is never disrupted, choppy, and uncoordinated. The world comes to me through my eyes as a continuous experience. I see all things from my 3Dⁱ perspective. I am a being that looks out on his three-dimensional world, with his eyes. By the way, you are too. I never experience anything that is 2D. If I pick up a piece of paper that is vertical and horizontal, it still has just the slightest breadth to it too. The paper is thin, but still has some depth to it. But, still we as 3D beings do peer in on a 2D world. I'm staring at a computer monitor. The words on this page are what a topologist would call an *embedded* dimension. That is, the 2D words on the monitor are inside my 3D world. Well if that's true, then what about my 3D world? Is it *embedded* in a larger 4D world? Topologically there is nothing to say it's not. What is really fascinating about this idea is this: We can't transcend dimensions? I can't touch or get into the words I'm writing now on this 2D screen. So, are there 4D beings beyond us that can't get into our worlds? If there are, what are they like? Would we be able to communicate with them if they exist across dimensional barriers?

A 4D world is one step higher than our 3D world. But, first lets examine the geometry that leads to our 3D worldⁱⁱ. So, look at the graphic below.



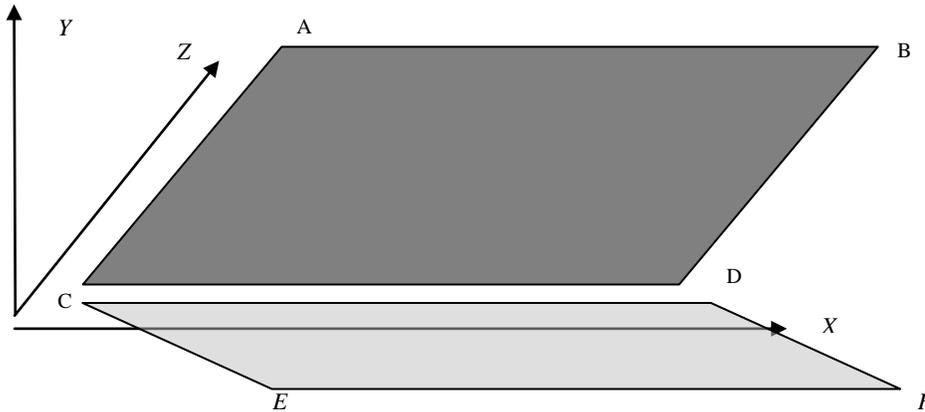
The 3 axes x , y and z give us all the dimensions we need for our 3D world. Now, if we wanted to create a 4D world, then a new axis called w would have to be introduced. For obvious reasons, I can't show you this 4th axis in a 2 dimensional graphic like above. But, I can show you how it must be generated in a 3D model. Notice that all of the 3 axes in the graphic above meet at 0. Each axis is at a 90° angle to the others. If our new dimension called w is to be successfully introduced to this axis system, it too will have to be at a 90° angle to the other 3. Lets set up some fundamental geometric concepts and then see if life forms could exist in these geometries.ⁱⁱⁱ

Points, Lines, Planes and Hyperplanes

In geometry we speak of dimensions or more rarely *dimensia* to mean classes of space. We can have $1,2,3...N$ dimensions. This means space itself can have infinitely many dimensions. There is an atomic unit called a *point*, which comprises all other objects in dimensions. A *point* is an assumed object that is not defined. A *point* has no dimensions--no length, width, or depth. *Lines* are made from at least 2 points. The 2 points must lie on *line* for the *line* to be drawn. This property is called collinearity. *Lines* unlike *points* do have a dimension. They have 1 dimension and this dimension can be extended infinitely. *Planes* are made when a *line* of 2 collinear points is extended to a third non-collinear point on another line. The plane will consist of at least 4 pairs of collinear points and will extend infinitely in 2 dimensions: length and height. And lastly a *hyperplane* is made when one plane with 4 collinear pairs is extended to a 5th non-collinear point of another plane. This construction will have 3 dimensions: length, height and width. As an example of a *plane* and a *hyperplane*, look at the two graphics below. Points A, B, C and D represent collinear points on 2 lines AB and CD



This plane has both length and height. It extends infinitely in both directions. Below is an example of a hyperplane in 3 dimensions.



Planes ABCD and CDEF intersect along line CD. These two planes together form a hyperplane in 3 dimensions. This hyperplane has length, height and width.

Lastly, a 4 dimensional plane can be formed from 6 points of a hyperplane (in the above case ABCDEF) and some point not in the dimensions they form (i.e. 3 dimensions). As stated above, I cannot draw a graphic of this figure in a 2 or 3 D space.

I've given a simple intuitive description of the geometry of dimensions. The same idea can be expressed in algebraic form. Although it's not necessary to understand multiple dimensions in an algebraic set theory sense, this model will have some bearing later when I discuss connected geometries. Thus, I must include it.

In algebraic set theory the *points* I defined above are really *ordered pairs*, such as (a,b). These *ordered pairs* are members of sets. The first member of an *order pair* is one dimension (x-axis) and the second member is the next dimension (y-axis). Say we have a collection of *ordered pairs* called set X (a,b). Take another collection of *ordered pairs* (c,d) called set Y. Sets X and Y can be related so that they form, lines, planes and hyperplanes. To see this, lets describe the hyperplane ABCDEF named above with algebraic set theory.

Let the points in ABCD name a set X

Let the points in CDEF name a set Y.

The symbol \rightarrow means *such that* | means *where*, \in means *member of* and, \times means *operated upon or related to*. We can now describe the graphics above in algebraic set theory notation:

The line CD on hyperplane ABCDEF is a subset of any ordered pair $CD \rightarrow X \cap Y$, that is, CD is the intersection of sets X and Y. The entire hyperplane is defined as follows: The hyperplane ABCDEF is merely the Cartesian product of all order pairs in ABCDEF. A Cartesian product is defined as follows: $X \times Y = \{(x,y) \mid x \in X \text{ and } y \in Y\}$. A Cartesian product uniquely maps every ordered pair in X to an ordered pair in Y, Thus ABCDEF hyperplane is the Cartesian product of $X \times Y$ and this amounts to being the union of sets X and Y.

Living Beings in Dimensional Spaces

Now that we have a framework of what 2, 3 and 4D spaces are, we can ask certain questions about living beings in these dimensions and their perceptions. I should be more precise. I can't really tell for sure what living beings in dimensions other than our own 3D realm would look like, but I can conjecture about what they could be and likewise conjecture exclusions on what they couldn't be.

2D, 3D and 4D beings

In the satirical little work *Flatland*, the author, Edwin A. Abbott, actually gives a good account of how beings in 2D would perceive their world. His account of 2D beings' bodies is flawed, though it's meant to be fictional. But that's not what we're going to consider yet. He correctly deduced that a 2D being would be only able to perceive lines of varying lengths. So, a triangle for us in 3D would appear to be a straight line for them. 2D beings (assuming there are such beings) can *perceive* only 2D objects embedded in a higher dimension, e.g. 3D. Going higher on the dimensional scale, we arrive at us: 3D beings. We seem to be able to perceive our world in full 3D mode. In fact, we perceive our world locally as flat, while it is curved. When we walk anywhere on this planet, even if we are climbing hills or mountains, we never perceive the terrestrial curve of the planet. We have to get in an aircraft and fly high off the surface of the planet to see the terrestrial curve. Even at this great height, we are STILL only seeing a curvature of a 3D object, i.e. Earth. Normally, we only see a uniform nearly flat surface in which we move, drive, swim, etc. In fact, due to our size compared to the size of the planet, we are never conscious of the speed that the globe is spinning (approximately 1000 miles an hour since the circumference of the planet is 24,000 miles). Moreover, we only perceive *objects* embedded in a higher dimension as 3D. What? I hear you say? What? Yes, as strange as that sounds, we are seeing 3D objects not 3D space! To illustrate my point in 2D first, lets take an example straight out of *Flatland*^{iv}. Abbott gives a diagram similar to the one below. Figure 1 is the view of an equilateral triangle from a 3D perspective, being flipped down. Figure 2 is a perspective from a 2D perspective.

Figure 1



Figure 2 _____

Notice in Figure 2 if we move the triangle around in the 2D it would not appear any different than a line.

If we expand this idea to 3D, we perceive all the embedded objects of a 4D dimension as 3D objects. But we can't see the dimension these objects are embedded in, e.g. the 4D. Physicists refer to the expansion of the universe in space-time. This *universal space-time expansion* is our universe expanding into a 4th spatial dimension. This is what we 3D beings can't see! So, here is the rule, though I will not attempt to prove it mathematically. Beings (if higher or lower dimensional beings exist) in one dimension can perceive only objects embedded in their current dimension or one below it. They never perceive the dimension itself.

TO BE CONTINUED

Ken Wais
4/13/09

ⁱ I will use the abbreviations 1D, 2D, 3D and 4D to mean respectively 1, 2, 3 or 4 dimensions.

ⁱⁱ In common parlance, we speak of higher and lower, when referring to abstract concepts that have nothing to do with these geometric terms. I will use these terms throughout this article to conform to popular usage that has made them readily understandable. However it should be noted that for instance, the 4th dimension is not 'higher' than the 3rd. A more precise term might be 'outside' or the adjective 'outer', but again these terms are not in common English usage when referring to the idea of dimension.

ⁱⁱⁱ A satirical book called *Flatland* by Edwin Abbott explores the idea of 2D beings embedded in a 3D world. I recommend it to anyone reading this article. Though much of the geometry it describes might contradict what modern mathematicians would accept, it aptly portrays the idea of living beings in another dimension.

^{iv} *Flatland*, page 48