Footnote

Since this essay is meant to be a non-technical exposition of Georg Cantor's Transfinite Theory of Infinite Sets, I have left out some very important details of how he proved that the Reals are larger than the Integers and thus Transfinite numbers.

What Cantor showed is that the Reals are uncountable as an infinite set. He did this using what is now called the diagonalization proof method. Below is the actual description of how Cantor developed his Diagonalization Proof of uncountable infinite numbers.

Look at the interval 0 to 1 of rational numbers, which are a subset of the reals. Within this interval we can form decimal expansions of each rational like such:

The rational numbers denote by Q can be expressed as the division of two integers like such: A/B. Between any two real numbers say 1 and 2 there are an infinite number of rational numbers in Q expressed in decimal form. *.x*. Rational numbers like *.x* have expansions that are infinite and countable. For instance, 1/4 which .25 is an infinite non-repeating expansion, like .250000000000000000000000….. ∞. The rational, 1/3 which is .333333333333333333….. ∞ is a repeating infinite expansion of 1/3. It is this subset of the reals that Cantor used to show that in the interval 0 to 1 of real numbers, we can have a set that is uncountably infinite. Let's take an example.

Remember I said you could specify a rule such that when two sets are related one set is guaranteed to be larger than another set? This is what we are going to do now in the way Cantor did it.

We will start with sets 4 of rational numbers, R_1,R_2, R_3, R_4that can be expressed in matrix form as follows:

$R_1 = |\, a_{ij} \,|$ Where *i* and *j* represent the row and column of each rational number in R_1. So, the 4 sets would look like this:

$R_1 = |\, 0.a_{11}, 0.a_{12}, 0.a_{13}, \ldots \,|$

$R_2 = |\, 0.a_{21}, 0.a_{22}, 0.a_{23}, \ldots \,|$

$R_3 = |\, 0.a_{31}, 0.a_{32}, 0.a_{33}, \ldots \,|$

$R_4 = |\, 0.a_{41}, 0.a_{42}, 0.a_{43}, 0.a_{44}, \ldots \,|$

If we consider these matrices as one matrix by adding each, R_n, this set would be a countable infinite set of rational numbers. But, suppose we wanted to a form a rational number that was not in this collection of sets? We could and furthermore we could use a

rule that ensured the rational number would not be in the collection of sets given, but would be uncountably infinite.

We can define a new rational number called X that has the following specification:

R_n =0. Whenever a_{ii}.= 1

And

R_n =1. Whenever a_{ii}.= 0

So, take our example above but with rational numbers in these matrices:

R_1=| 0.010 …. |

R_2=| 0.101 …. |

R_3=| 0.. 101 |

R_4=| 0.. 1011

In this case the new rational number X would be:

X =.1100 ($a_{11,}$ a_{22} a_{33} a_{44}) That is when $a_{11,}$=0 then R_n = 1

And we could go on making a new rational number X as long as we had R_n numbers and this number would not be in the set of rational numbers generated by R_n numbers. It would an uncountable set of rational numbers.